BEHAVIORAL MODELS OF USERS IN RIDE-SHARING

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1 Introduction

Commercial ridesharing, wherein on-demand service providers such as Lyft and Uber offer "pooled" versions of their ride-hailing services in real time and users share rides with other users, has emerged as a popular solution to potentially combat ever-increasing road congestion. However, recent data from the City of Chicago shows that only about 25% of all passengers *chose* to share a ride with another passenger. Moreover, roughly 30% of such passengers actually ended up traveling alone, because the service provider did not find a good match (see Table 1). Furthermore, these ridesharing systems are quite complex, and consist of several key elements that impact outcomes, which can be classified as follows:

1. *Operational* modules include matching, routing, and fleet/supply management.
2. *Economic* modules include pricing, compensation, and incentive design.
3. *Behavioral* modules include modeling user choice/response and service quality (QoS).

Table 1: Month-to-month breakdown of percentage of passengers who chose shared, from Chicago Data Stats (Nov 2018 - July 2019). Total Number of Passengers: 73,247,231. Number of Passengers Chose Shared: 18,348,891 (25%). Number of Passengers Actually Shared: 12,946,164 (17.7%).

Month	Percentage	Month	Percentage
Nov 2018	26.8%	Mar 2019	25.3%
Dec 2018	26.5%	Apr 2019	24.6%
Jan 2019	28.1%	May 2019	22.5%
Feb 2019	26.8%	Jun 2019	19.8%

In this paper, we focus only on the behavioral module in a real-time setting: a commercial service provider offers both exclusive and shared rides, and will set the corresponding prices depending on the estimated additional delay/inconvenience for a shared ride. We develop a random utility model for users, a discrete choice model on these utilities, and a notion of QoS that incorporates reference effects and loss aversion into the traditional concept of ex-post individual rationality.

2 Methodology: User Behavior Model Design

Users interact with the service provider through an interface on a mobile device, similarly to popular ridesharing services. The interaction consists of the following two stages:

1. **Stage 1:** User j inputs their source coordinate (S_j) and destination coordinate (D_j), and receives a menu of service options. For simplicity, we assume just two options—an exclusive ride with no detours at an upfront price of p_j^x, and a (possibly) shared ride with an estimated detour of $\hat{\delta}_j$ at an upfront price of p_j^s.

2. **Stage 2:** The user evaluates these options and performs one of three actions: requesting the exclusive service, requesting the shared service, or neither. Under the former two actions, the user is assigned an appropriate vehicle, which could result in either initiating a new ride (possible under either service), or modifying an existing ride (possible only under the shared service).

In the first state, when a user j inputs their source and destination coordinates, the service provider considers all possible existing shareable rides that could *feasibly* detour from their existing route to serve this additional user. Then, for each of these feasible rides, the provider computes the optimal values of p_j^x, p_j^s, and $\hat{\delta}_j$, as well as the corresponding optimal incremental profit, if user j were to be added to these rides (Biswas et al. 2017). The ride offering the maximum optimal incremental profit is then chosen as a *tentative* match, and the corresponding optimal prices and detour estimate are returned to the user. For any two spatial coordinates A and B, we let $d(A, B)$ denote the shortest distance from A to B. An exclusive ride always provides service along a shortest route.

2.1 User's Utility:

User i has a valuation $v_i > 0$ per mile for exclusive service. These valuations are independently and identically distributed across users, according to a distribution with cumulative distribution function F_v and corresponding density function f_v. For shared service, the user's valuation depreciates by a factor $k_i(\delta_i)$, a decreasing function of δ_i, the *fractional* detour experienced by user i. To be precise, δ_i is the *additional* distance travelled by user i due to sharing service (over and above the shortest distance $d(S_i, D_i)$), as a fraction of $d(S_i, D_i)$. We let $k(0) = \overline{k} \leq 1$ to model fixed, non-detour-related inconveniences from sharing. Thus, the utility function of user i is given by:

$$U_i(\texttt{choice}_i; p_i^x, p_i^s, \delta_i) = \begin{cases} v_i d(S_i, D_i) - p_i^x, & \texttt{choice}_i = \texttt{Exclusive} \\ k_i(\delta_i) v_i d(S_i, D_i) - p_i^s, & \texttt{choice}_i = \texttt{Shared} \\ 0, & \texttt{choice}_i = \texttt{Declined} \end{cases} \tag{1}$$

At the time of making the choice, user i does not know the actual detour δ_i that they would experience. Instead, they only know the estimated detour $\hat{\delta}_i$. Thus, the users set $\texttt{choice}_i$ to maximize $U_i(\texttt{choice}_i; p_i^x, p_i^s, \hat{\delta}_i)$. Moving forward, we define

$$\hat{U}_i^s(p_i^s) = U_i(\texttt{Shared}; \cdot, p_i^s, \hat{\delta}_i) = k_i(\hat{\delta}_i) v_i d(S_i, D_i) - p_i^s \tag{2}$$

to be the *estimated* utility of user i when choosing `Shared`, and

$$U_i^s(p_i^s, \delta_i) = U_i(\texttt{Shared}; \cdot, p_i^s, \delta_i) = k_i(\delta_i) v_i d(S_i, D_i) - p_i^s \tag{3}$$

to be the *actual* utility of user i when choosing `Shared`.

We assume that the depreciation functions k_i of the users and the distribution F_v of their exclusive per-mile valuations are known to the service provider; however, the realized valuations v_i are private information to the users.

2.2 User's Choice:

Intuitively, users with 'low' v_i would choose `Declined`, those with 'high' v_i would choose `Exclusive`, and those with 'intermediate' v_i would choose `Shared`. We now formalize this threshold behavior of the user choice.

If a user i chooses `Shared`, then, it implies that $\hat{U}_i^s(p_i^s)$ (defined in (2)) is greater than the utility from choosing `Exclusive` or `Declined`:

$$k_i(\hat{\delta}_i) v_i d(S_i, D_i) - p_i^s > \max\{0, v_i d(S_i, D_i) - p_i^x\}. \tag{4}$$

Simplifying the above inequality yields

$$\underline{v}_i < v_i < \overline{v}_i, \tag{5}$$

where the lower and upper bounds, $\underline{v}_i$ and $\overline{v}_i$ are given by:

$$\underline{v}_i = \frac{p_i^s}{k_i(\hat{\delta}_i) d(S_i, D_i)}, \qquad \overline{v}_i = \frac{p_i^x - p_i^s}{(1 - k_i(\hat{\delta}_i)) d(S_i, D_i)}. \tag{6}$$

An immediate necessary condition for (5) to be satisfied for some v_i is that $\underline{v}_i < \overline{v}_i$, which yields:

$$p_i^s < k_i(\hat{\delta}_i) p_i^x, \tag{7}$$

which imposes a constraint on the prices that the service provider considers, should it be feasible to offer a shared ride option to user i. Moreover, when the above constraint is violated, i.e., when $p_i^s \geq k_i(\hat{\delta}_i) p_i^x$, the exact value of p_i^s does not affect a user's choice between `Exclusive` and `Declined`, since that choice would be completely determined by p_i^x. This observation relieves the service provider from explicitly considering $p_i^s > k_i(\hat{\delta}_i)p_i^x$, simplifying the search space. Thus, without loss of generality, we assume that

$$p_i^s \leq k_i(\hat{\delta}_i)p_i^x. \tag{8}$$

A similar analysis for the choices `Exclusive` and `Declined`, under (8), results in the following user choice function:

$$\texttt{choice}_i^*\left(v_i;\underline{v}_i,\overline{v}_i\right) = \begin{cases} \texttt{Declined}, & v_i \leq \underline{v}_i \\ \texttt{Shared}, & \underline{v}_i < v_i < \overline{v}_i \\ \texttt{Exclusive}, & v_i \geq \overline{v}_i \end{cases} \tag{9}$$

2.3 A New User's Impact on an Existing Shareable Ride:

Suppose a new user j is being considered for addition to an existing shareable ride with $j-1$ users in the vehicle. (We assume that $j-1 \geq 1$; the bootstrapping problem of computing optimal prices to be offered to a user to initiate a new shareable ride does not involve any existing passengers.) Without loss of generality, we assume that the indices of the existing users are in the order in which they are scheduled to be dropped off according to the existing route plan, with ties broken arbitrarily (e.g., when two or more existing users share a common destination). Let D_0 denote the current location of the vehicle. If user j is added to the existing ride, we assume that the new route plan leaves unchanged the relative order in which the existing users are scheduled to be dropped off. (This enables the routing optimization to be quick, by limiting to a quadratic number of possibilities.) Let $t_j^- < j-1$ (respectively, $t_j^+ \leq j$) denote the largest (respectively, smallest) among the indices of existing users who are dropped off immediately before picking up (respectively, after dropping off) user j, according to the new route plan. If nobody gets dropped off before j is picked up, define $t_j^- = 0$. (If j is the last user to be dropped off, define $t_j^+ = j$.) Define the following quantities:

$$\Delta_j^s = d\left(D_{t_j^-}, S_j\right) + d\left(S_j, D_{t_j^-+1}\right) - d\left(D_{t_j^-}, D_{t_j^-+1}\right) \tag{10}$$

$$\Delta_j^d = \begin{cases} d\left(S_j, D_j\right) + d\left(D_j, D_1\right) - d\left(S_j, D_1\right), & t_j^+ = 1 \\ d\left(D_{t_j^+-1}, D_j\right) + d\left(D_j, D_{t_j^+}\right) - d\left(D_{t_j^+-1}, D_{t_j^+}\right), & 1 < t_j^+ < j \\ d\left(D_{j-1}, D_j\right), & t_j^+ = j \end{cases} \tag{11}$$

Δ_j^s and Δ_j^d are the *source detour* and *destination detour* from the current route to serve the additional user j, respectively. Not all existing users experience both of these detours, as we discuss next.

Suppose δ_i^{j-1} denotes the fractional detour that would be incurred by an existing user $i < j$ according to the existing route plan, if user j is not added to the shared ride. Then,

$$\delta_i^j = \delta_i^{j-1} + \frac{\mathbb{1}\{i > t_j^-\}\Delta_j^s + \mathbb{1}\{i \geq t_j^+\}\Delta_j^d}{d(S_i, D_i)} \tag{12}$$

is the fractional detour that would be incurred by user i according to the new route plan if user j is added to the shared ride. Let δ_j^j denote the fractional detour that user j would experience according to the new route plan.

2.3.1 Individual Rationality (IR):

A shared ride is individually rational (IR) for a user, if their utility from the shared ride is nonnegative. There are different notions of IR in the literature; the one we focus on is called *ex-post IR*, and means that the *actual* utility of the user at the end of the shared ride, given by (3), is nonnegative, that is, $U_i^s(p_i^s, \delta_i) \geq 0$ for all i. Since U_i^s is a decreasing function of δ_i, this property is always satisfied when the service provider ensures that $\delta_i \leq \hat{\delta}_i$ for all i, because that would, in turn, ensure that $U_i^s(p_i^s, \delta_i) \geq \hat{U}_i^s(p_i^s)$, which is nonnegative because the user chose `Shared`.

Motivated by recent experiments (Cohen et al. 2018) highlighting the benefits to a service provider of *proactively* compensating users whose ex-post IR constraints may have been violated, we adopt ex-post IR as an indicator of the provider's Quality-of-Service (QoS).

2.3.2 Sequential Individual Rationality (SIR):

Our notion of *sequential IR* (SIR) requires that the service provider *sustain* ex-post IR for all the users, at every stage of a shared ride. In other words, whenever a new user j is considered for addition to an existing shareable ride with $j-1$

users in the vehicle, the service provider must ensure that $U_i^s(p_i^s, \delta_i^j) \geq 0$ for all $i \leq j$, where δ_i^j is given by (12). At first glance, such a notion may seem unnecessarily strong; however, in our model, SIR is *necessary* to ensure ex-post IR. This is because, for a fixed p_i^s that was committed to at the time that user i joined the shared ride, $U_i^s(p_i^s, \delta_i)$ is a decreasing function of δ_i, which, in turn, is nondecreasing as new users are added.

This concern about violating ex-post IR was empirically addressed by Cohen et al. (2018) in their experimental study. Their findings motivate one to consider both the discrete choice model, and the consequences of violating SIR in the interim (during the shared ride) but restoring ex-post IR (at the end of the shared ride) of each user through an appropriate monetary compensation to the user (penalty to the provider).

3 Results: Numerical Examples

We illustrate the spatial properties of the optimal shareable region visually, using a small numerical example in the Euclidean space $\mathbb{R}^2$. First, we consider a simple scenario where all the users are traveling to a common destination, D. A shared ride is 'bootstrapped' by the first passenger, whose source is S_1. The grey shaded ellipse-shaped region in Figure 1 (*left*) depicts the "SIR-feasible region" $\mathcal{R}_j^{SIR}(D)$ (for $j = 2$), the collection of source coordinates S_2 from which a second user can be added to the ride without violating SIR for the first user. In the same figure, the dashed curves represent the boundaries of the optimal shareable regions when the service provider is sensitive to the QoS (sensitivity captured by scalar β, the optimization problem is omitted for brevity). Then, Figure 1 (*center* and *right*) shows how these regions change as the ride progresses, for $j = 3$ and $j = 4$, respectively, for a random selection of S_2 (and subsequently S_3).

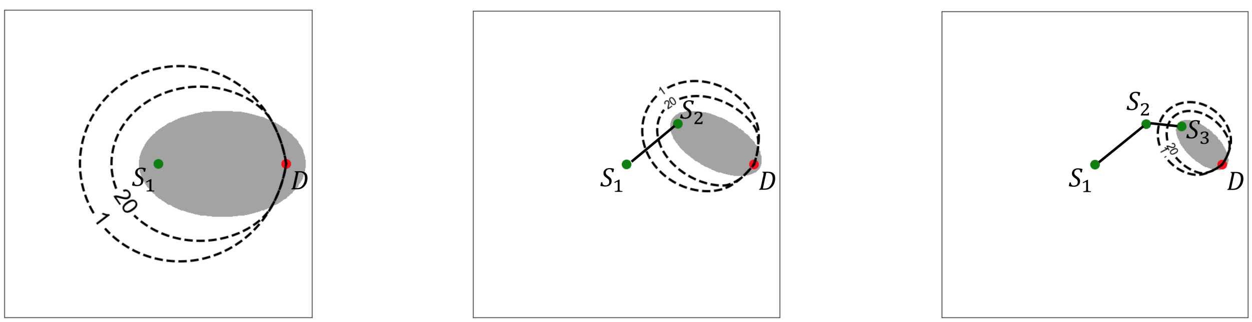

Figure 1: Evolution of the optimal shareable region (interior of the dashed curves, for $\beta = 1, 20$) from within which it is profitable to add a subsequent user to the ride, as the ride progresses and more users are added. For reference, the grey shaded area shows the SIR-feasible region.

First, observe that there are points within $\mathcal{R}_j^{SIR}(D)$ that are not in the optimal shareable region. Thus, even though the provider incurs no penalty by adding a user from such points, it would be suboptimal to do so. Next, observe that the portion of $\mathcal{R}_j^{OPT(\beta)}(D)$ that is outside $\mathcal{R}_j^{SIR}(D)$ is smaller for $\beta = 20$ than for $\beta = 1$ (service provider is less sensitive to user's QoS in the latter case).

Next, we consider a more complex scenario where users have different sources and destinations. In Figure 2, the interpretations of the dashed curves and the grey shaded regions are the same as before, except that they depict possible locations of D_2 (*left*) and D_3 (*right*), respectively. The bottom half of Figure 2 is the "zoomed out" version of the top half, that demonstrates that $\mathcal{R}_j^{OPT(\beta)}(S_j)$ (for $j = 2, 3$) are *closed* regions. The shapes that define the regions in Figure 2 are more complicated than those in Figure 1 due to the spatial discontinuities associated with the order in which the users are dropped off.

4 Conclusion

We proposed a model of user behavior and a notion of service quality in ride-sharing, which can be considered while designing the economic and operational modules to improve the utilization numbers stated above. For instance, a QoS-sensitive service provider's profit optimization problem can be formulated by internalizing the users' choices, taking into account any penalties for QoS violations. Our system specific modeling of user behavior could potentially be generalized to be more broadly applicable to dynamic shared service systems in which the quality of shared service, as perceived by customers based on their experience (e.g., waiting in queues, delays/interruptions during service), plays a central role in designing operational and economic policies (e.g., matching/routing, staffing, pricing).

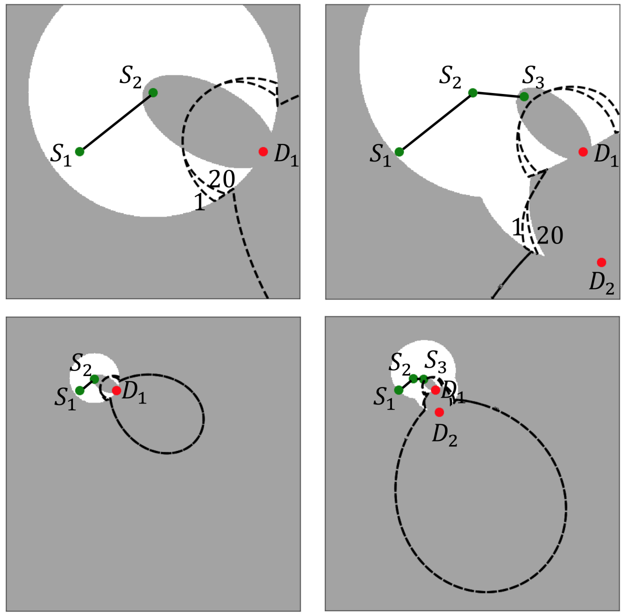

Figure 2: Evolution of the optimal shareable region (interior of the dashed curves, for $\beta = 1, 20$) to within which it is profitable to drop off the subsequent users, as the ride progresses and more users are added. For reference, the grey shaded area shows the SIR-feasible region.

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